Side channel attacks based on linear approximations

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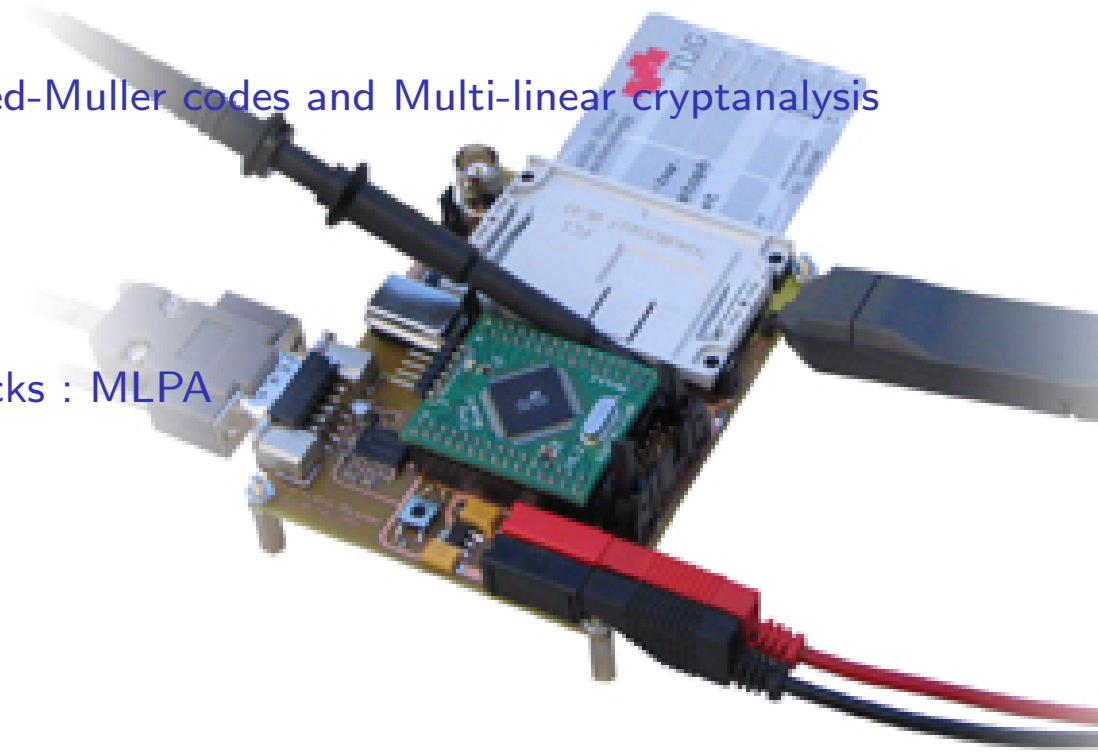
Outline



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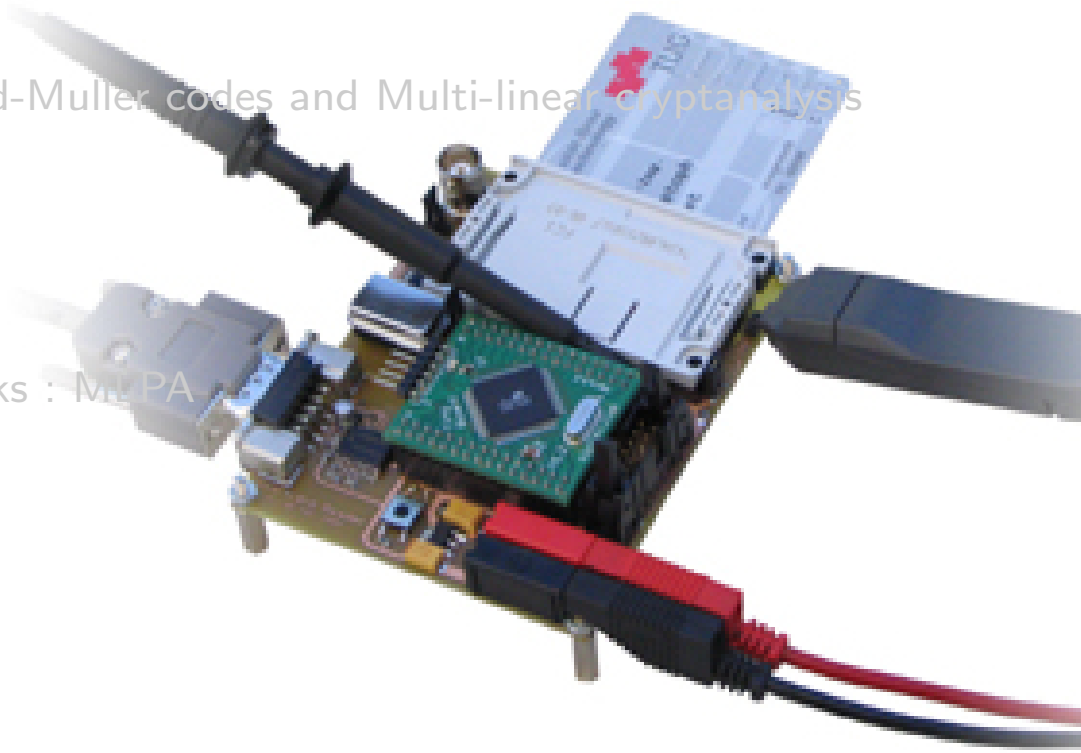
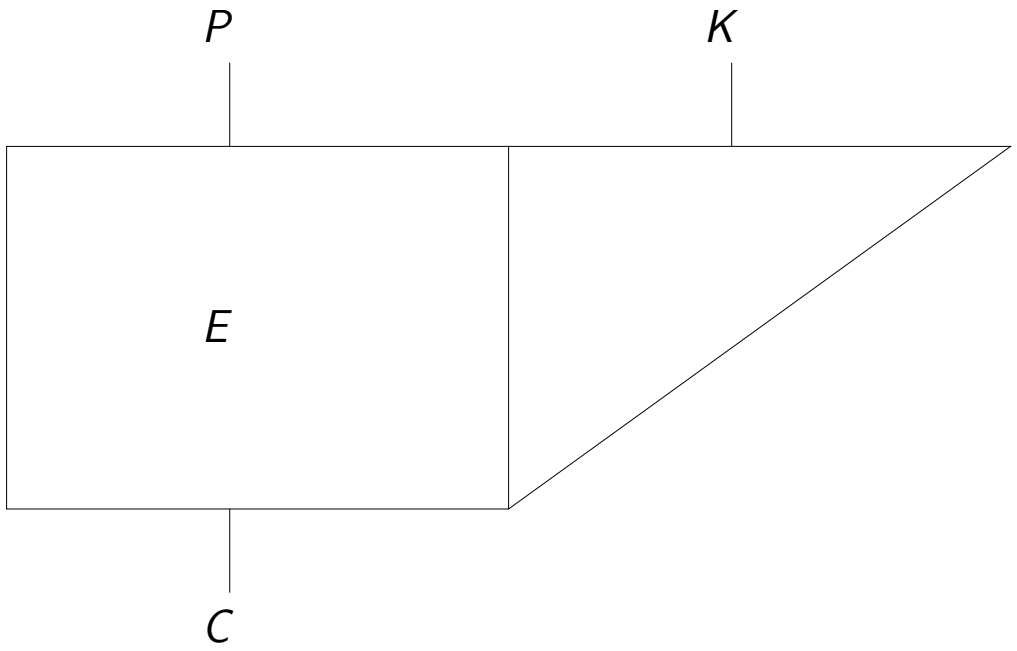


Block cipher



Iterated block cipher : A Mathematical object

- DES (1974)
  - $\mathcal{P}$: 64 bits
  - $\mathcal{K}$: 56 bits
  - best attack $\sim 2^{56}$ (brute force)
- AES (2000)
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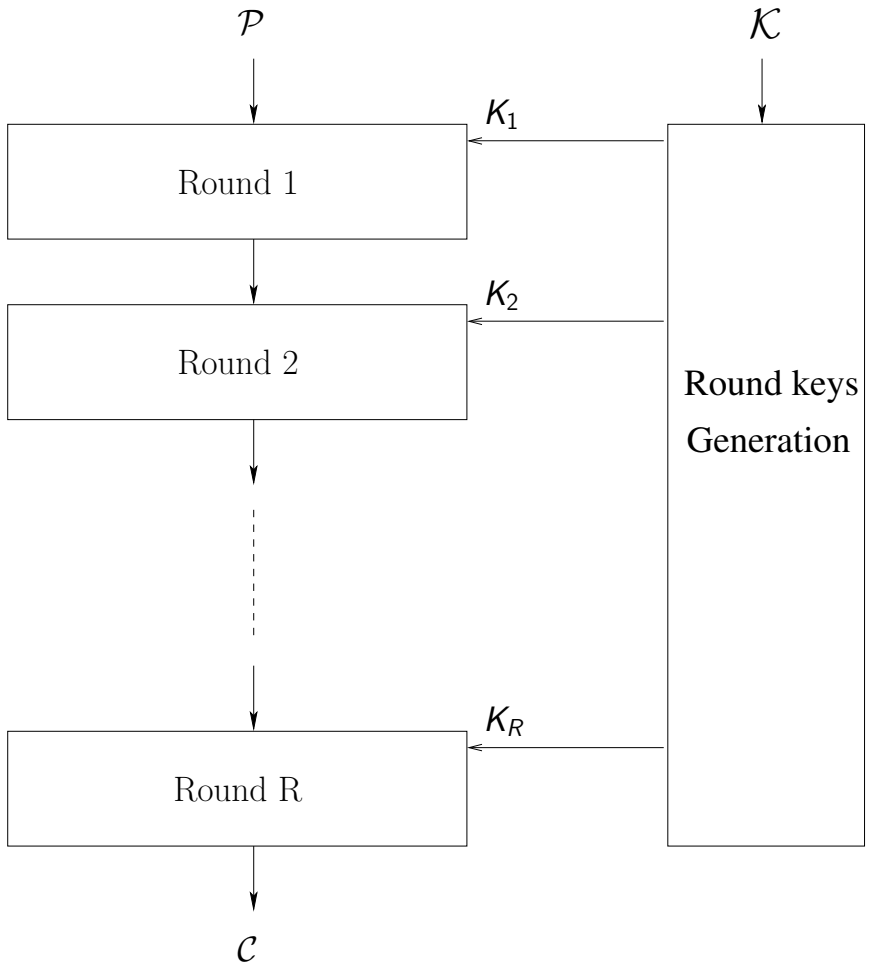
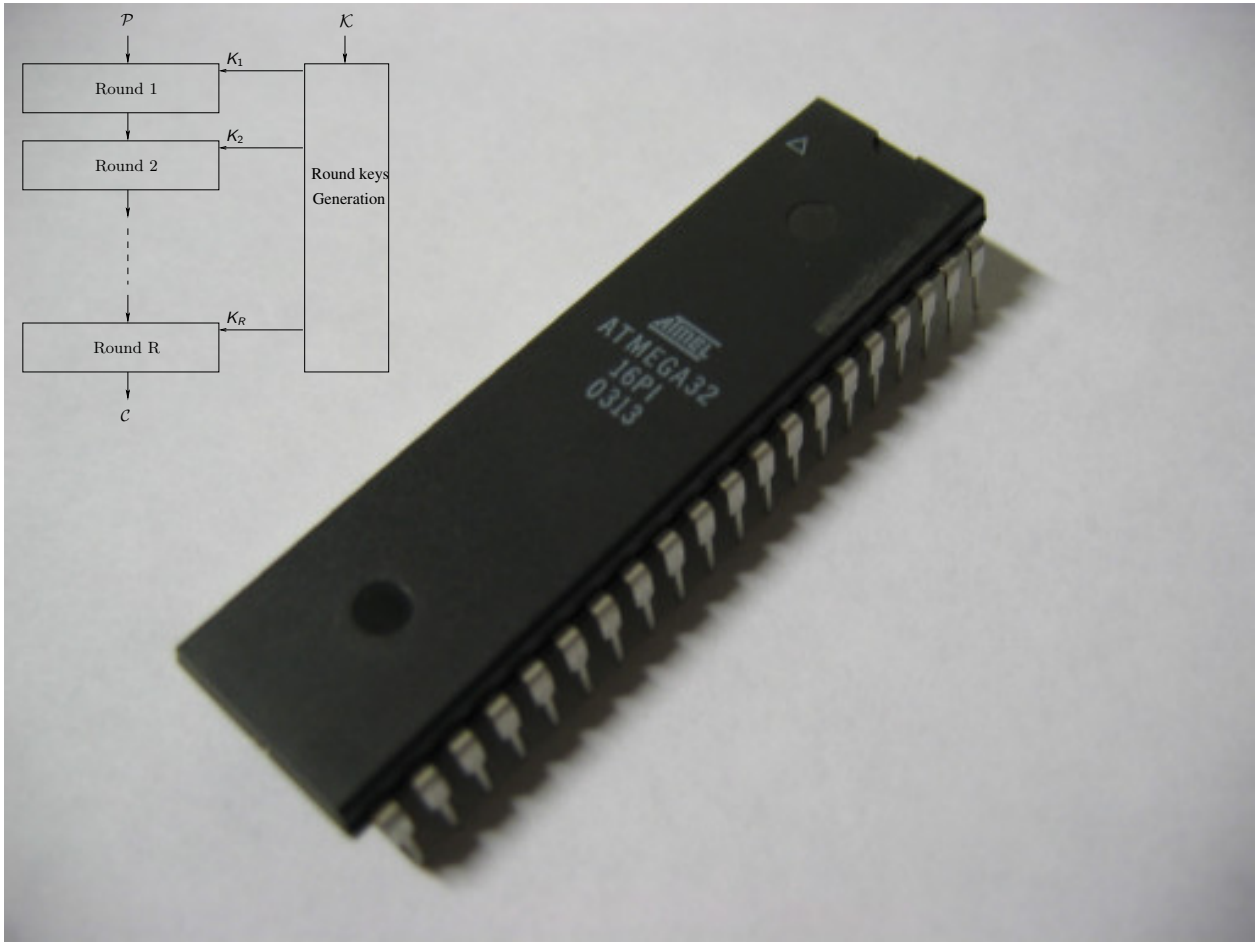
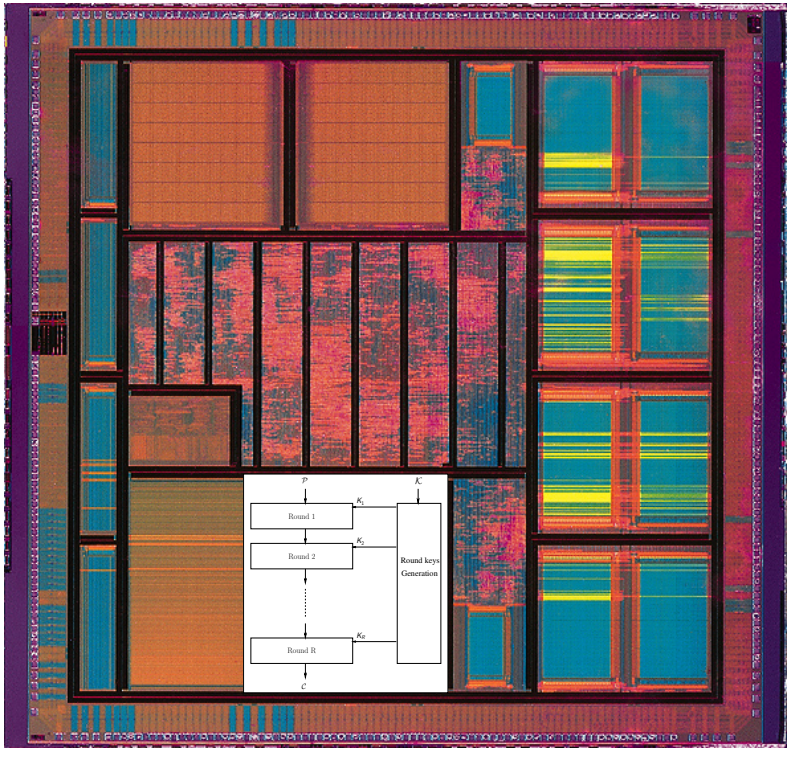


Iterated block cipher : Software Implementation

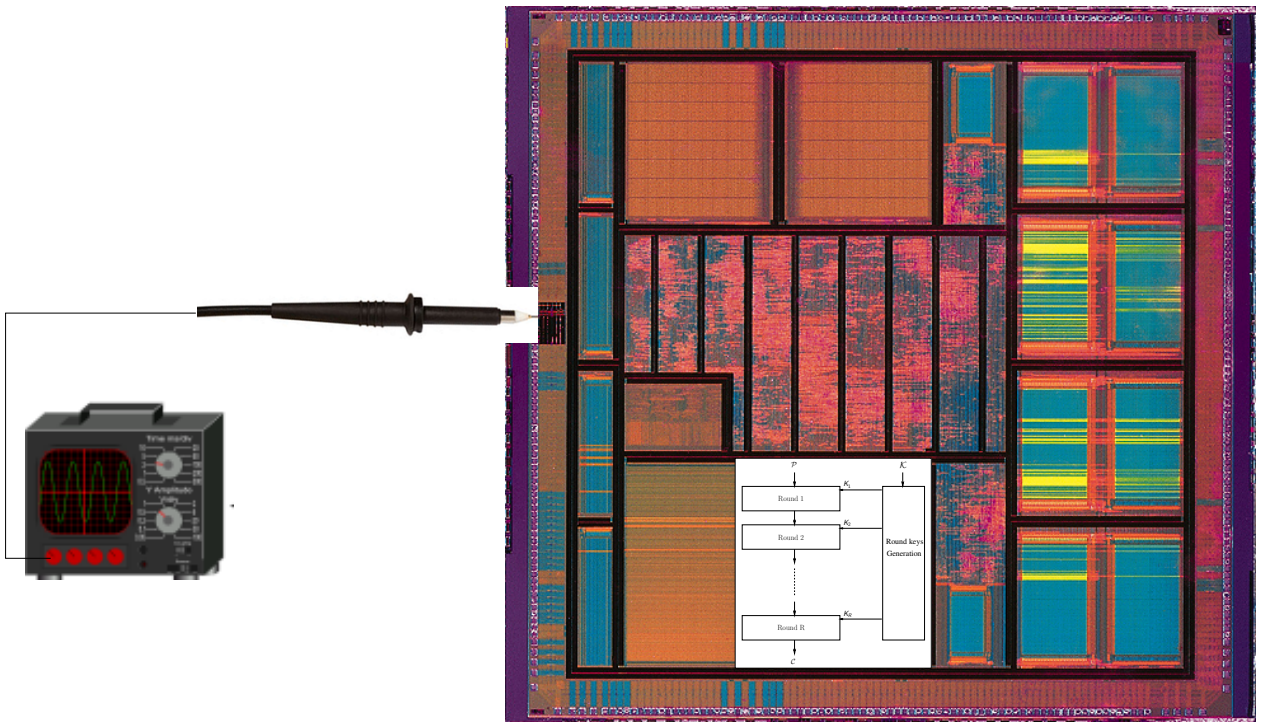


Iterated block cipher : Hardware Implementation

- Hardware : ASIC, FPGA
  - asynchronous vs synchronous implementation
  - iterative vs unrolled implementation



Side channel attack using consumption leakage



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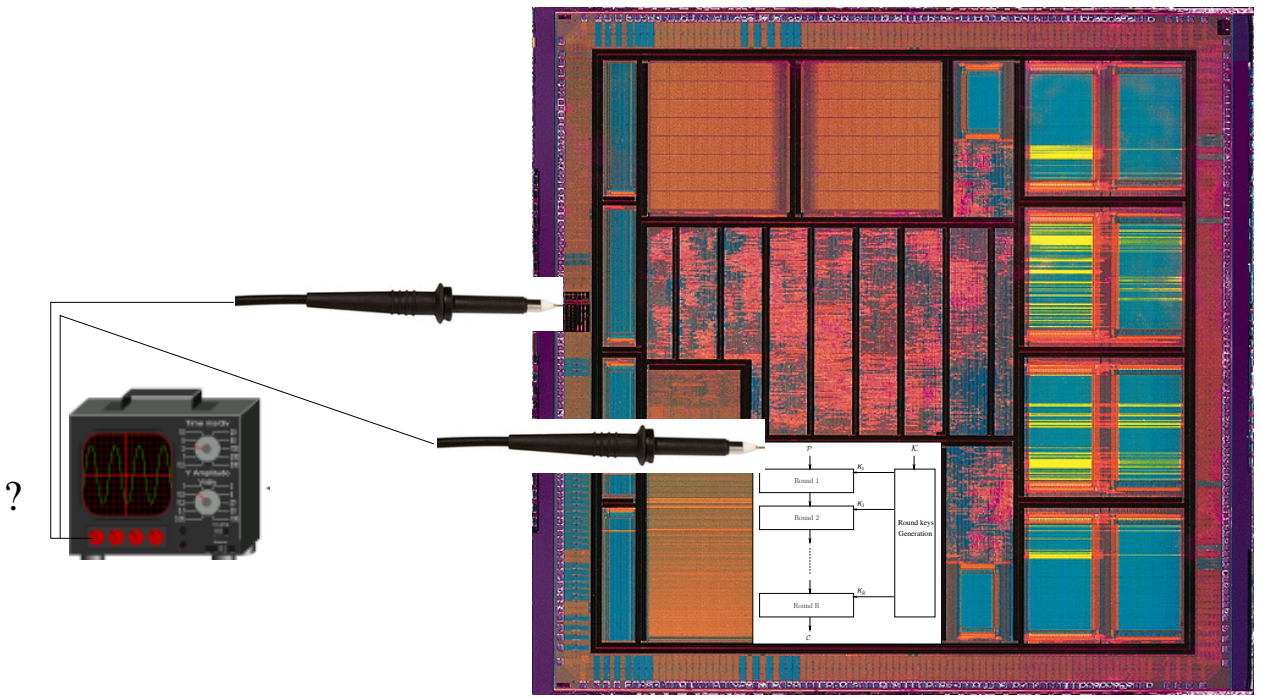
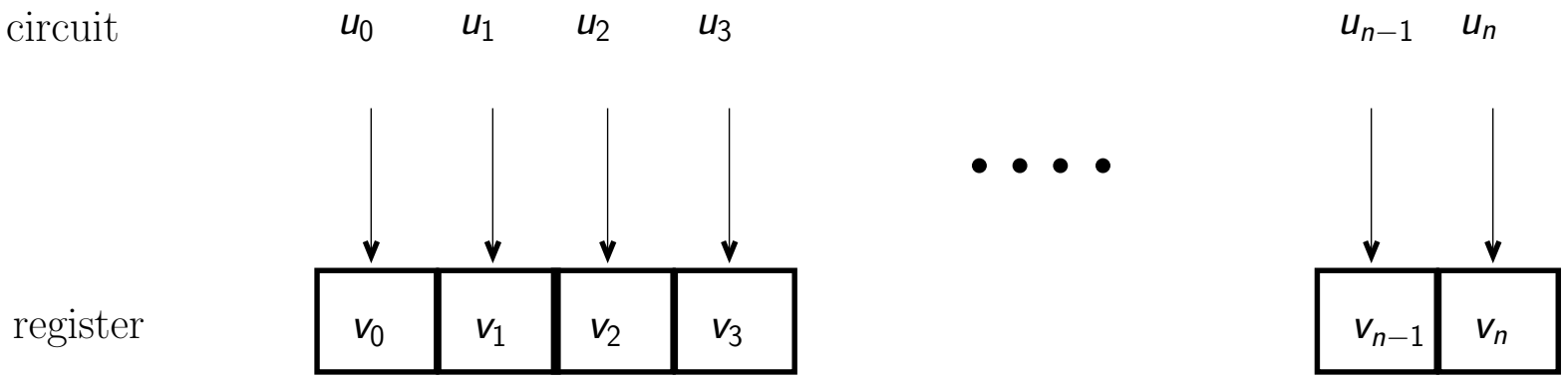


Side channel attack using consumption leakage

Consumption Model

A CMOS transistor consumption is high during a bit-flip otherwise it is negligible.



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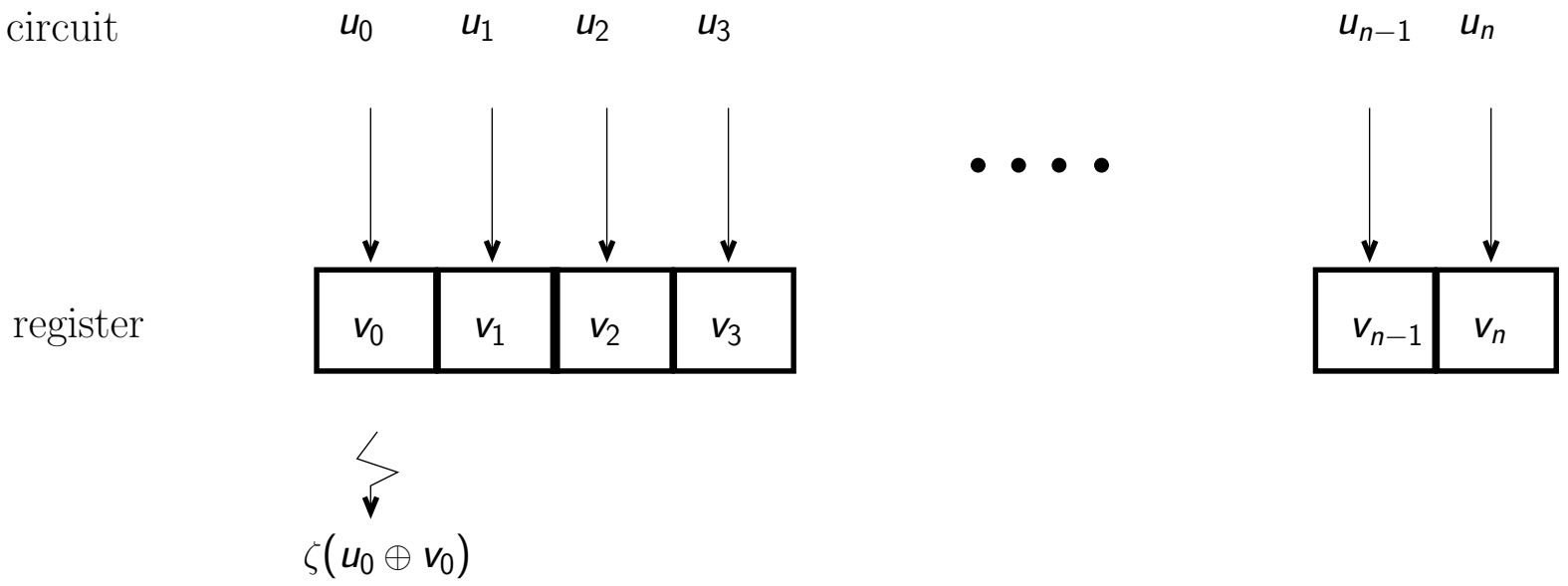

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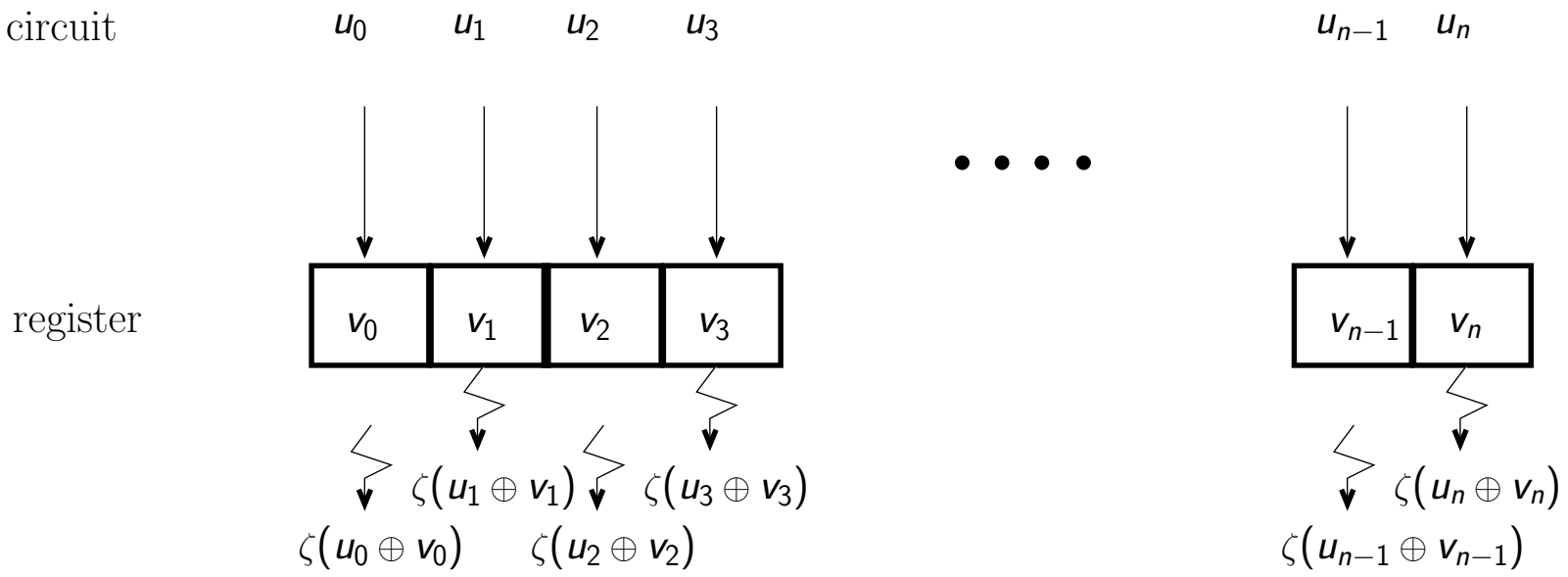
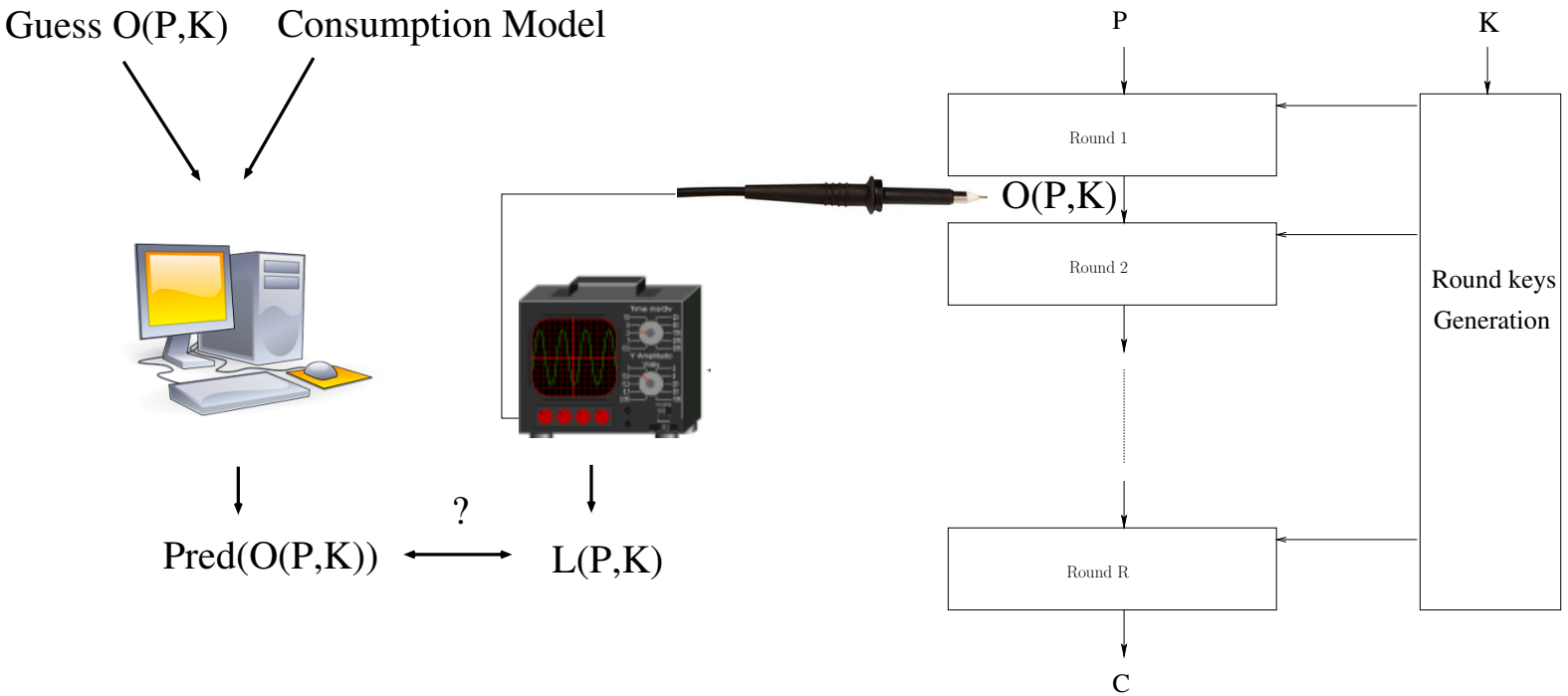


$$\text{Consumption} \sim \zeta \times HW(u \oplus v)$$

Classical Power Analysis Attacks (DPA [Kocher 98])



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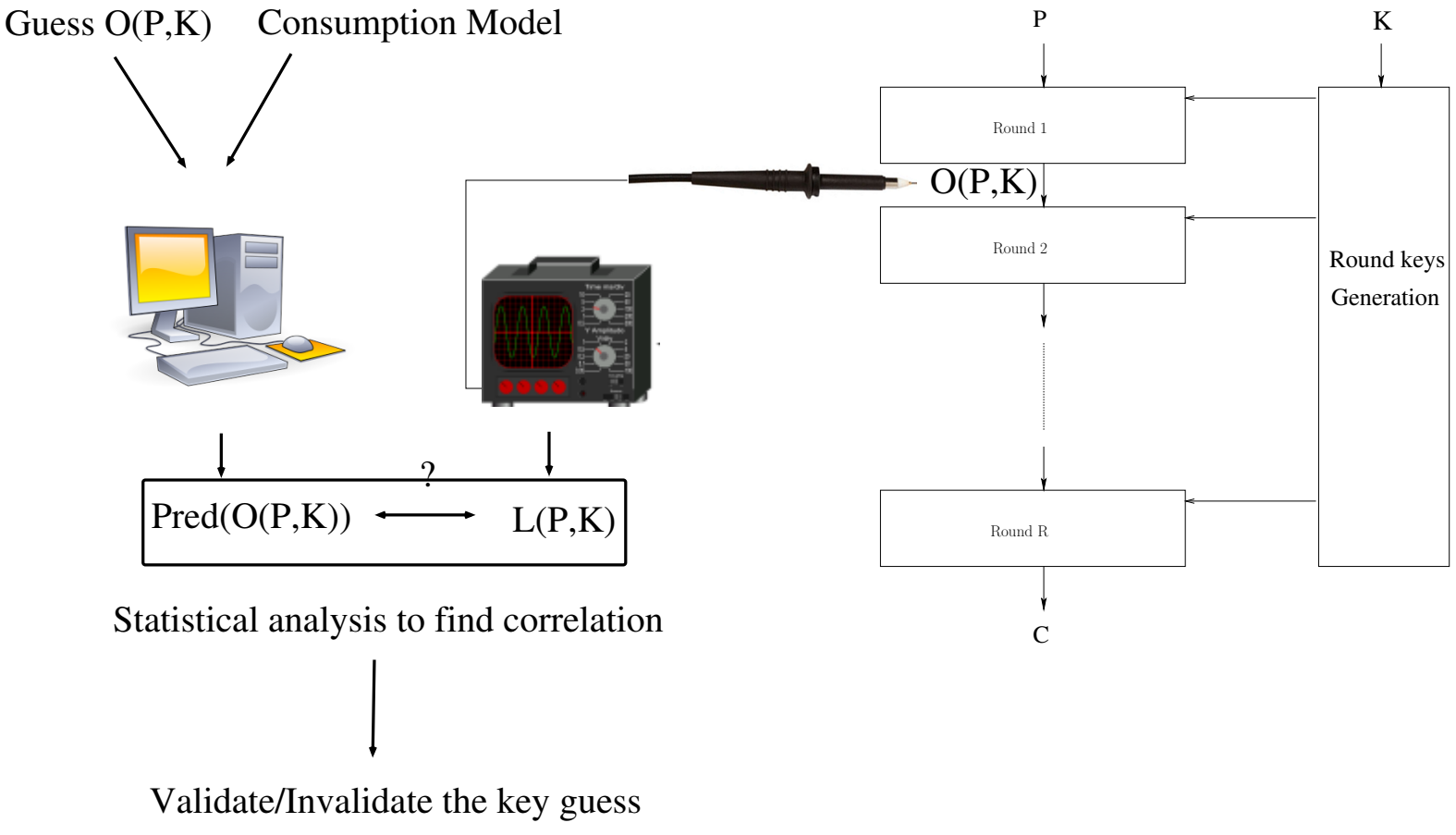

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Countermeasures

The implementation is safe if one can shut down all information leakages :

Suppress synchronization elements. (buses and registers)

and/or Randomize the data processed.
(Random Masks [Goub 99, Char 99, Akka 01, Lv 05, Riva 08])

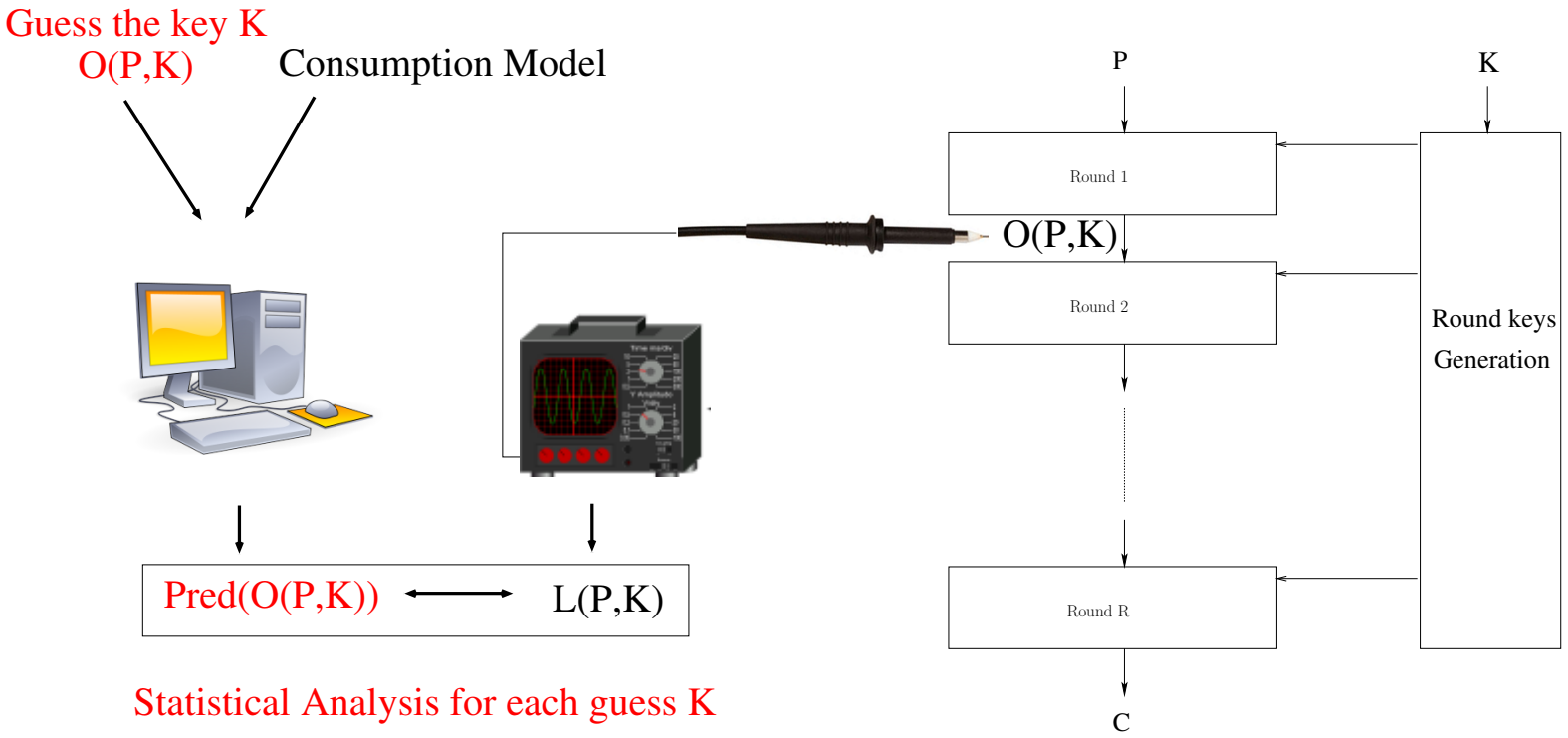
and/or Add random useless computations.

and/or balanced dynamic dual-rail gates designs.

and/or ...

The sound countermeasures are very expensive !

Limitations of classical Power Analysis Attacks



Restriction to intermediate values $O(P, K)$ dependent to less than 32 key-bits.
i.e. first and last round of the cipher.

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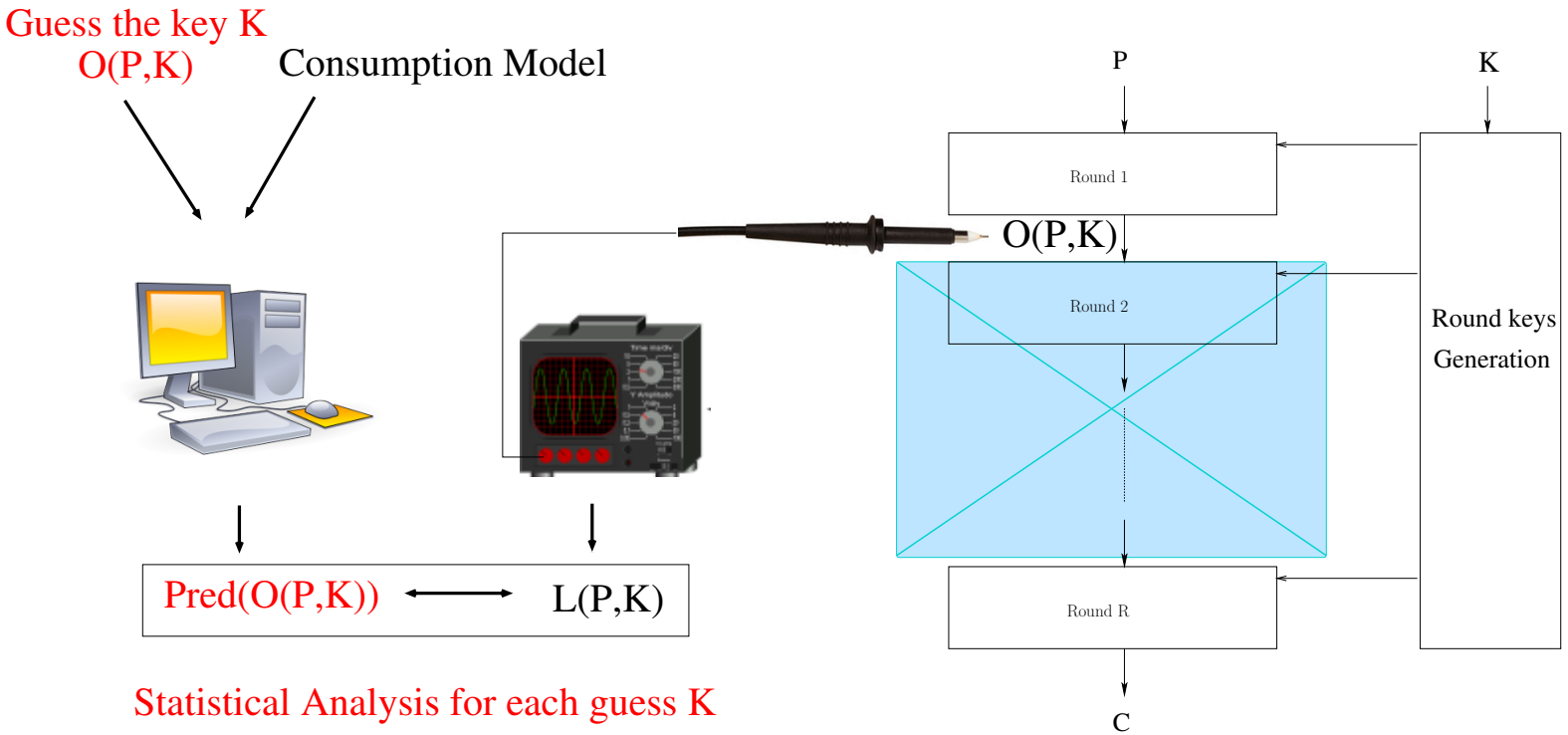

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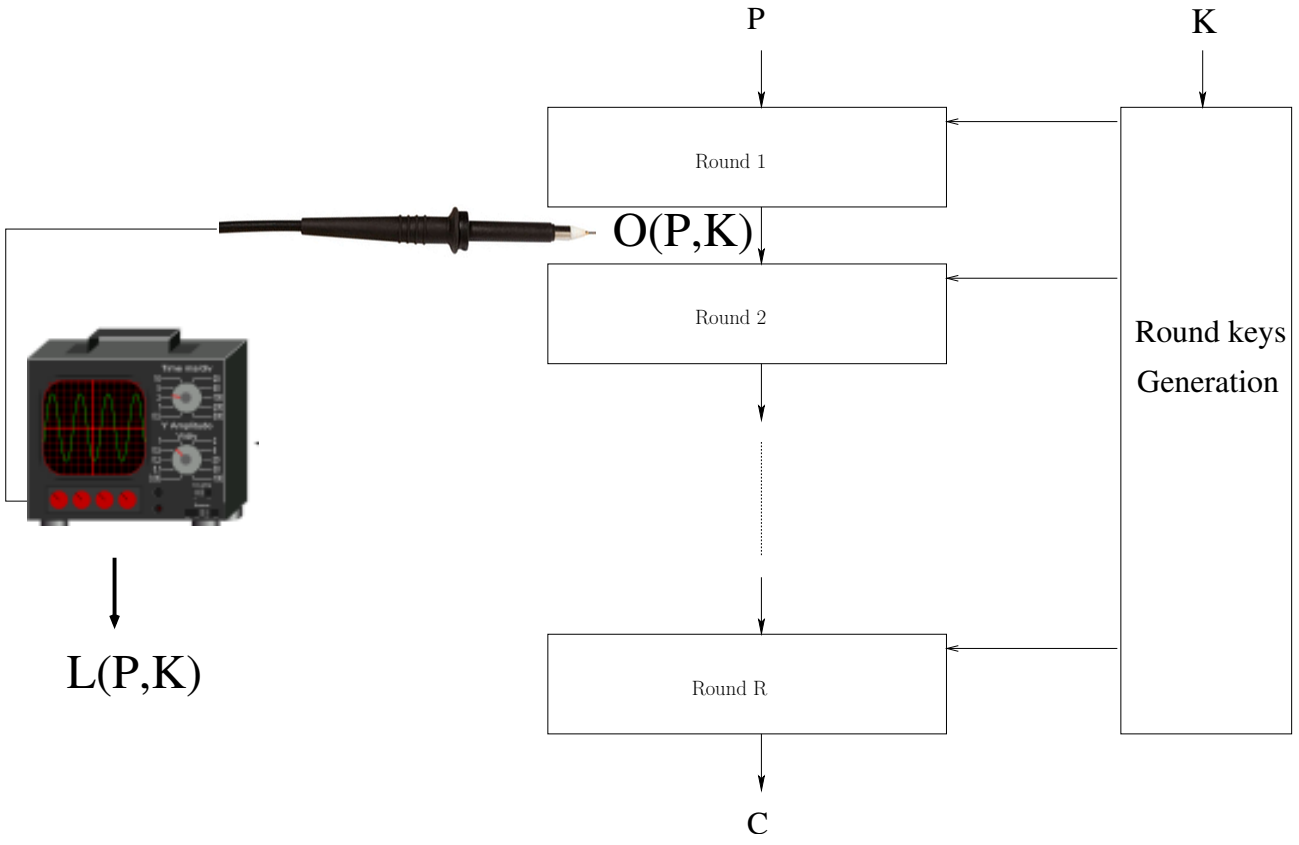
We want to use deeper leakage points



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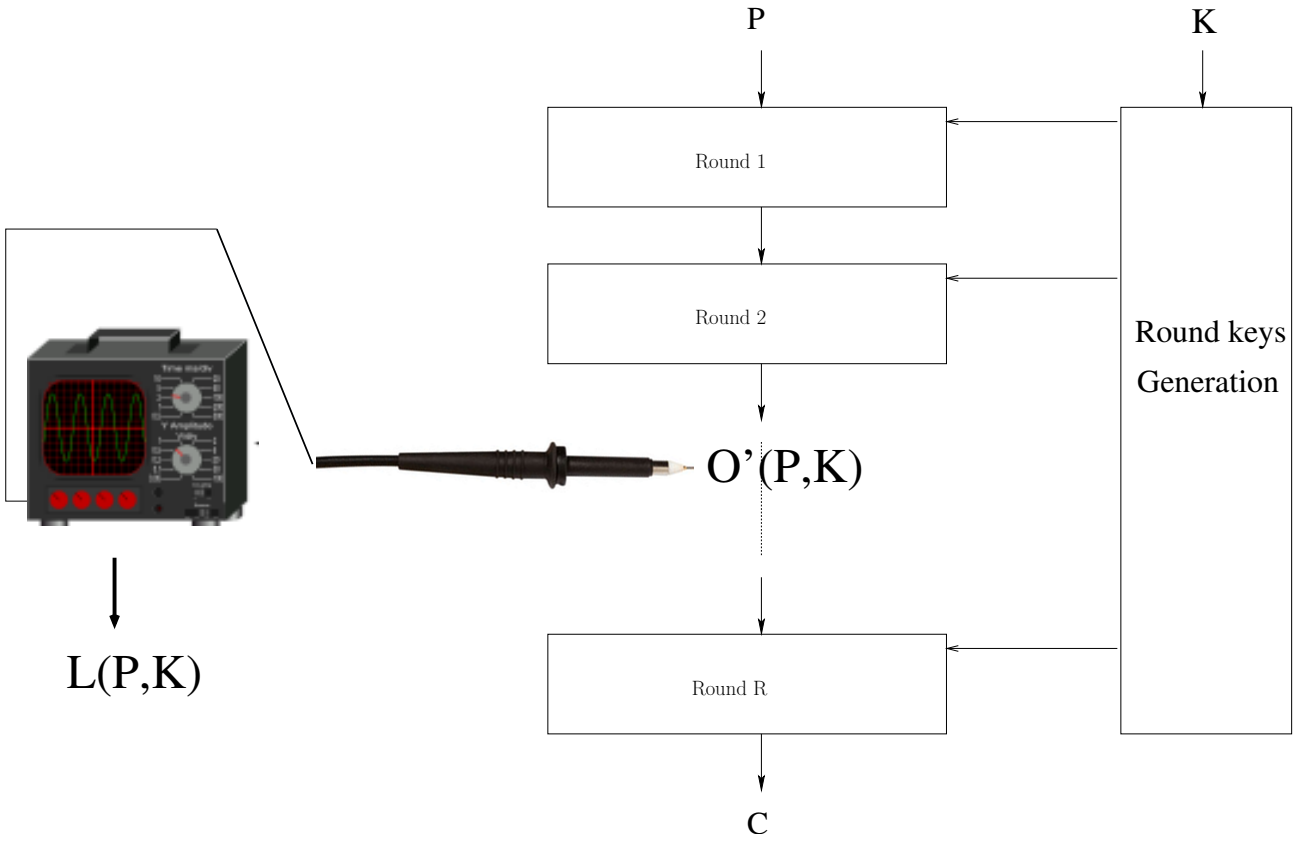


Glued Blocks

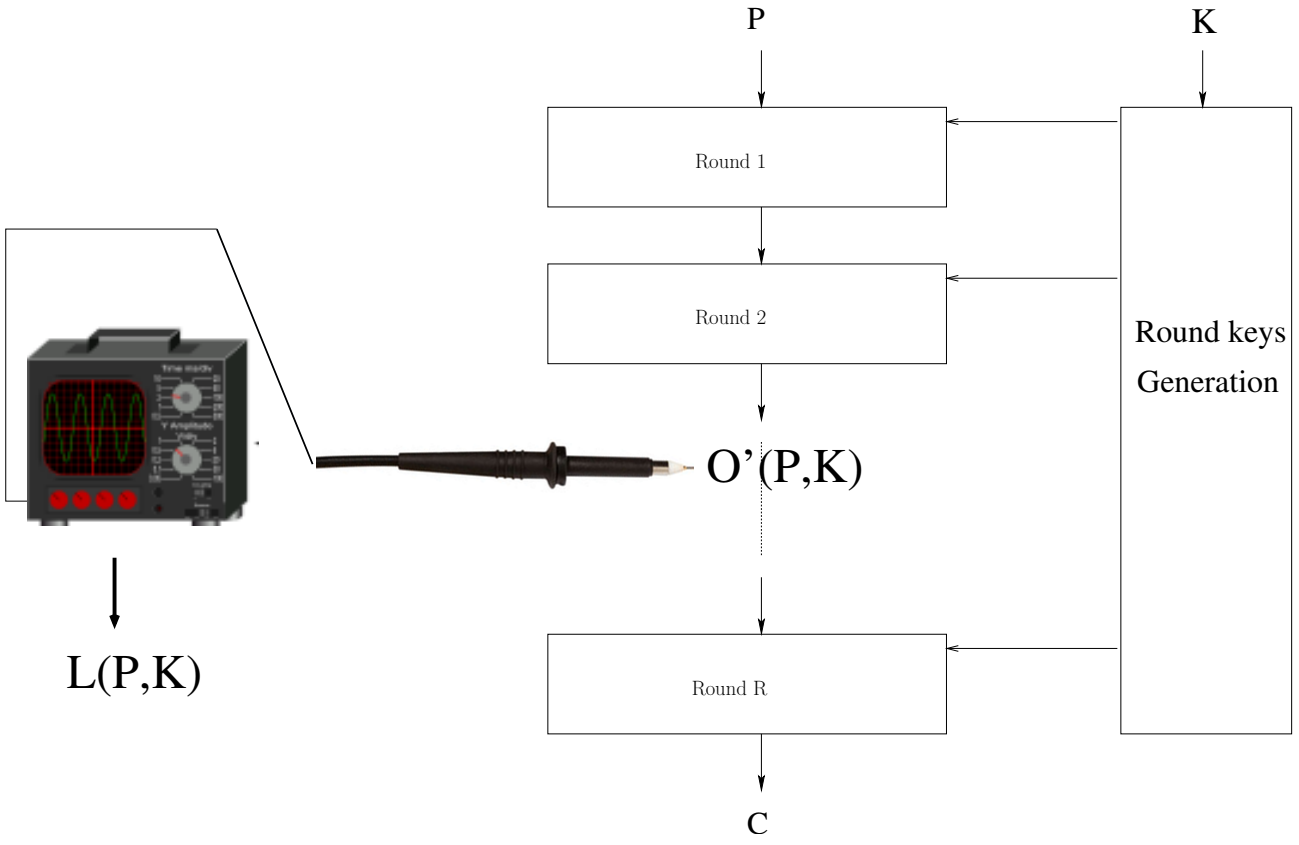
light countermeasures [Akkar Goubin 03]

Concentrate countermeasures on the firsts and lasts rounds.
i.e. no information leak during these critical rounds

$\Rightarrow$ No observable intermediate value is dependent to less than 32 key bits.

Sufficient countermeasure against classical power analysis attacks : DPA, CPA, MIA, etc ...

Simultaneous multiple side channel measurements



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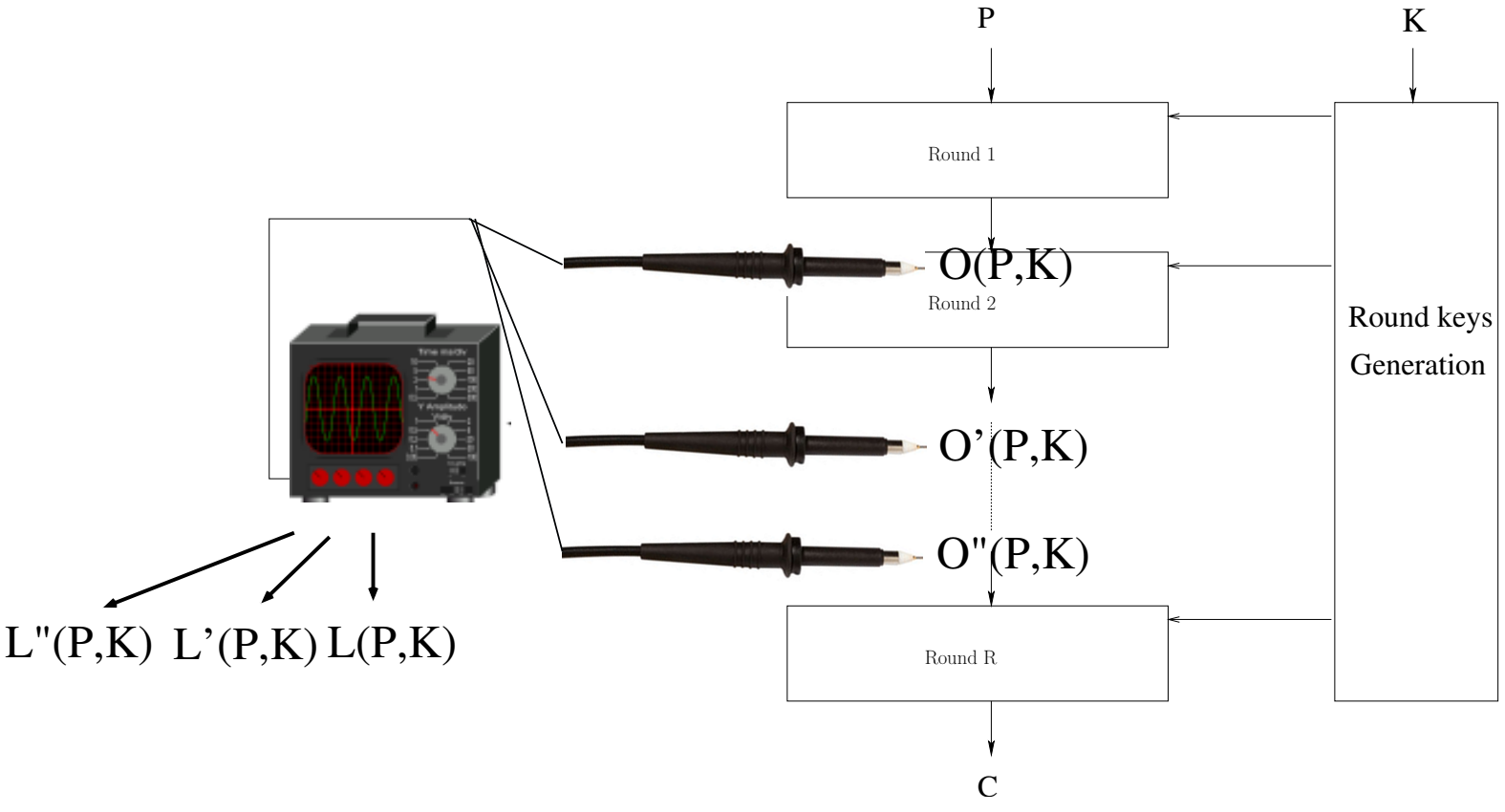


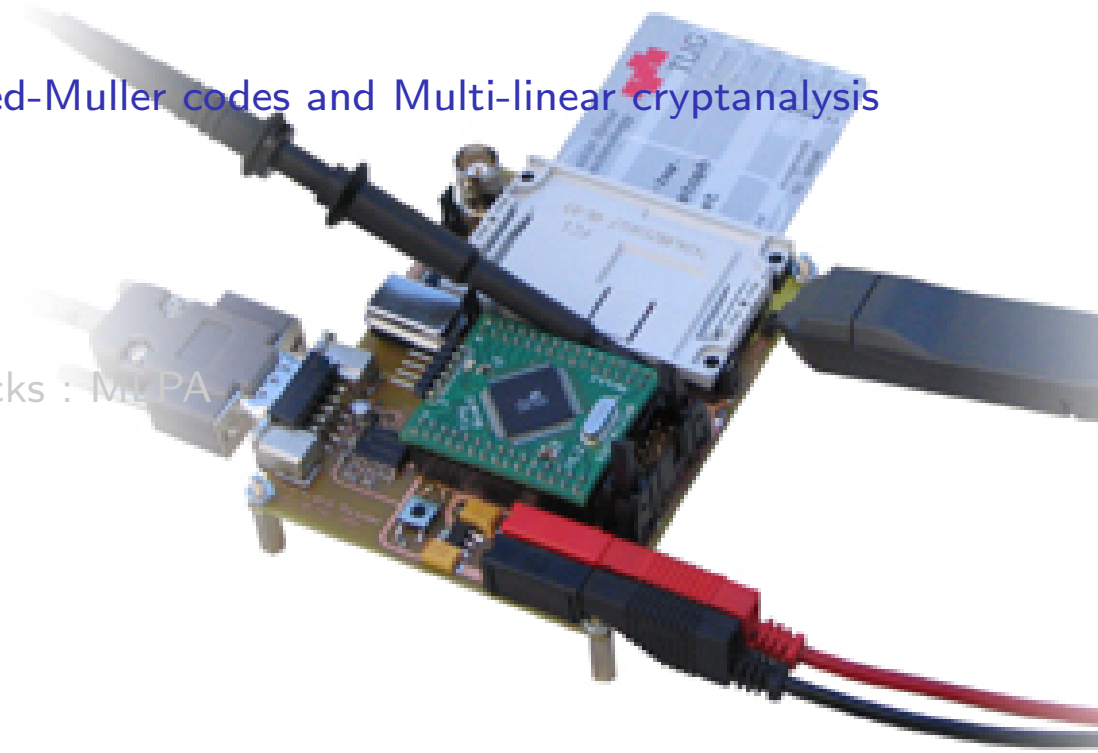
Related work

- Differential-based side channel [Carl 05, Hand 06]
- Collision-based side channel
  [Schr 03, Schr 04, Ledi 04, Bogd 07, Bogd 08b, Bogd 08a]
- Algebraic side channel [Rena 09]

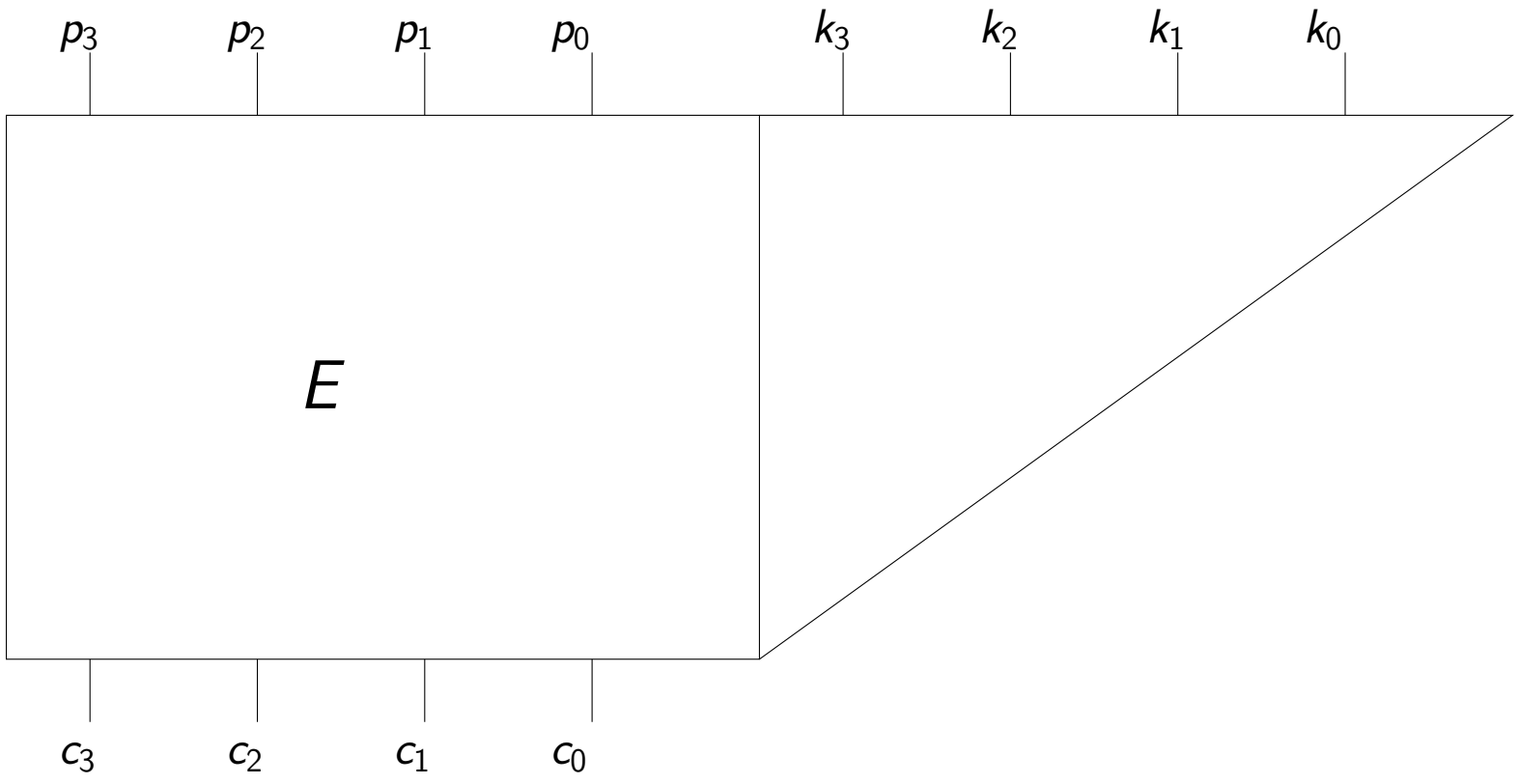
Limitation

These method usually need to have very precise measures (template-like attacks).
Hypothesis : The attacker must be able to precisely evaluate $O(P, K)$ from $L(P, K) \Rightarrow$ Access to a training board.

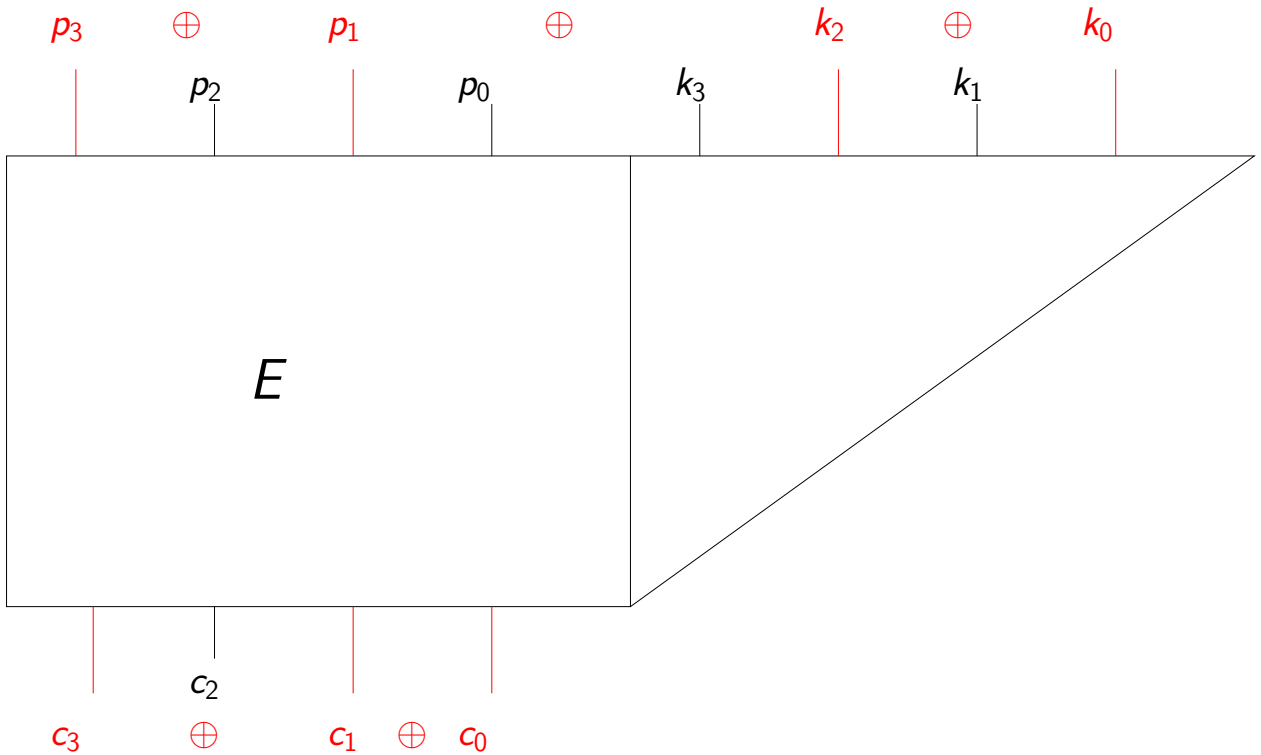
Outline



Symmetric cipher (4-bits plaintexts, 4-bits key)



Linear approximations



linear approximation

$p_1 \oplus p_3 \oplus k_0 \oplus k_2 = c_0 \oplus c_1 \oplus c_3$
hold with probability $p = 1/2 + \epsilon$.

Multi-linear cryptanalysis

$$\left\{\begin{array}{lcl|l} k_0 \oplus k_2 & = & p_1 \oplus p_3 \oplus c_0 \oplus c_1 \oplus c_3 & p = 1/2 + \epsilon_1 \\ k_0 \oplus k_1 \oplus k_2 & = & p_0 \oplus p_2 \oplus c_2 \oplus c_3 & p = 1/2 + \epsilon_2 \\ k_1 \oplus k_3 & = & p_2 \oplus p_3 \oplus c_1 \oplus c_3 & p = 1/2 + \epsilon_3 \\ k_1 \oplus k_2 \oplus k_3 & = & p_0 \oplus p_1 \oplus p_2 \oplus p_3 \oplus c_2 \oplus c_3 & p = 1/2 + \epsilon_4 \end{array}\right.$$

Complexity of the attack [Biry 04]

Given n linear approximations $\langle \alpha_i, P \rangle \oplus \langle \mu_i, K \rangle = \langle \beta_i, E(P,K) \rangle$

$$\#Plaintexts = O(\frac{1}{\sum_i (\epsilon_i^2)})$$

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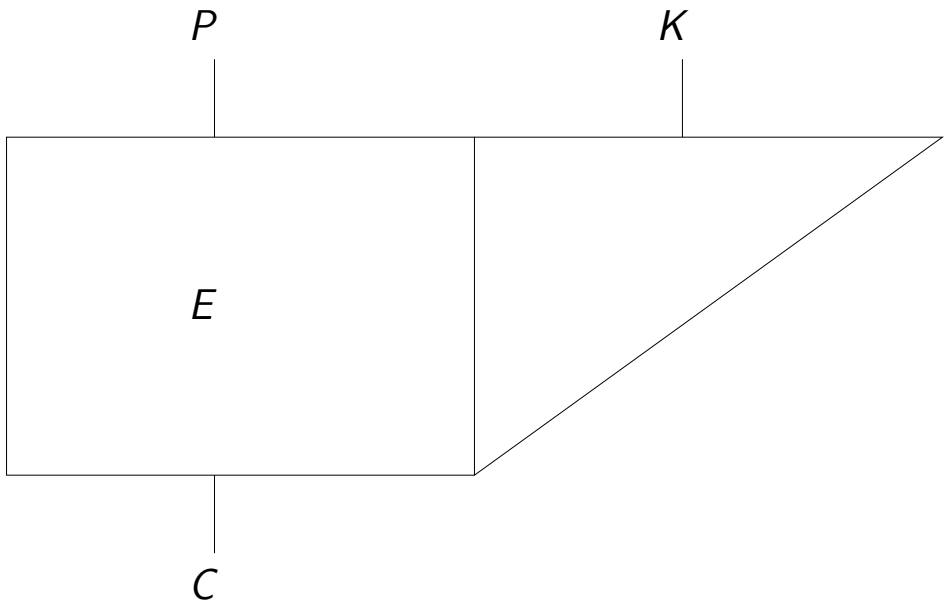
Multivariate degree 1 polynomial reconstruction

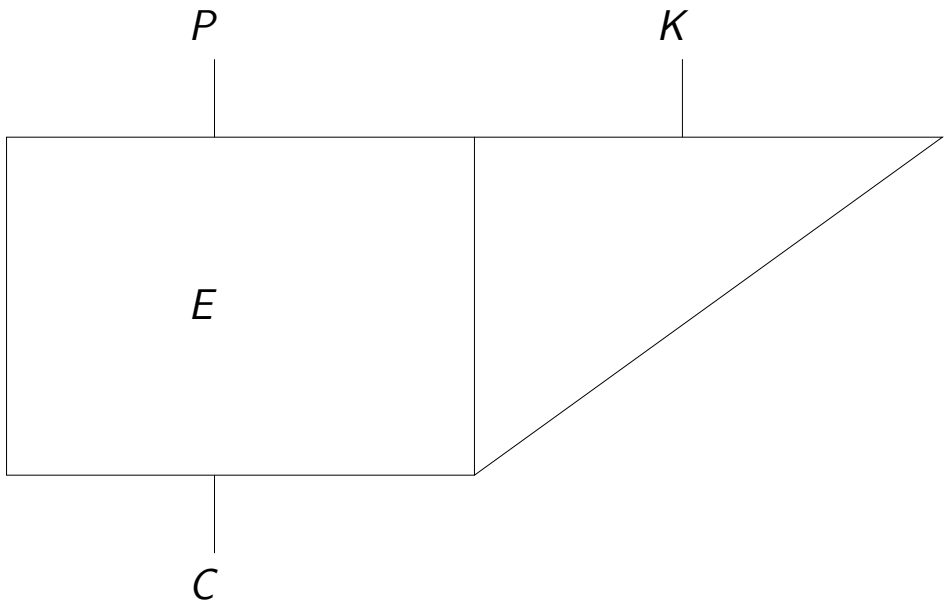


$$\langle \alpha, P \rangle \oplus \langle \mu, K \rangle = \langle \beta, E(P, K) \rangle$$

An interpretation of the problem

By fixing β, we fall in a problem list decoding of the first order Reed-Muller code, we have to decode the noisy codeword $\langle \beta, E(P, K) \rangle$.

Multivariate degree 1 polynomial reconstruction



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Reed-Muller code properties

Definition of $RM(1, m)$

- $RM(1, m) = \{f \in GF(2)^{(1)}[x_1, x_2, \cdots, x_m]\}$;
- Usual representation : $(f(0), f(1), \cdots, f(2^m - 1))$;
- Boolean representation : $f = f_1 x_1 \oplus f_2 x_2 \oplus \cdots \oplus f_m x_m$
- code of length $n = 2^m$ and minimal distance $d = n/2$.

Classical Problem

Given a Boolean function g, we want to construct the list $\{f \in RM(1, m) \mid d_H(f, g) \leq n(1/2 - \epsilon)\}$, which is equivalent to $L_g(\epsilon) = \{f \in RM(1, m) \mid I^{(g)}(f) = \sum_{x \in GF(2)^m} (-1)^{f(x) \oplus g(x)} \geq 2\epsilon n\}$.

Johnson Bound

In fact $\|L_g(\epsilon)\| \leq \frac{1}{4\epsilon^2}$

List Decoding Algorithms

A simple idea

$2\epsilon n \leq |l^{(g)}(f)| \leq \sum_{s \in GF(2)^{m-i}} |\sum_{r \in GF(2)^i} (-1)^{g(r,s) \oplus f^{(i)}(r)}|$ where

$f^{(i)} = f_1 x_1 \oplus \cdots \oplus f_i x_i$.

Screening process : we suggest f_i and we check if the inequality is satisfied.

$\Rightarrow L_g^{(i)}(\epsilon) = \{f \in RM(1,i) \mid \sum_s |\sum_{r \in GF(2)^i} (-1)^{g(r,s) \oplus f(r)}| \geq 2\epsilon n\}$.

Complexity

Worst case complexity

The complexity of this algorithm is in $\mathcal{O}(n \log_2^2(\epsilon))$ [I Du 07].

The complexity of the prob. version is in $\mathcal{O}(m^2/\epsilon^6)$ [Kaba 04].

The size of the result can be of size $m/2\epsilon^2$, thus optimal complexity could be in $\mathcal{O}(m/\epsilon^2)$.

Optimal complexity

In fact Goldreich and Levin algorithm : $\mathcal{O}(m/\epsilon^4)$.

I. Dumer, G. Kabatiansky and C. Tavernier [Four 09] :

$$\mathcal{O}(m/\epsilon^2)$$

Conclusion

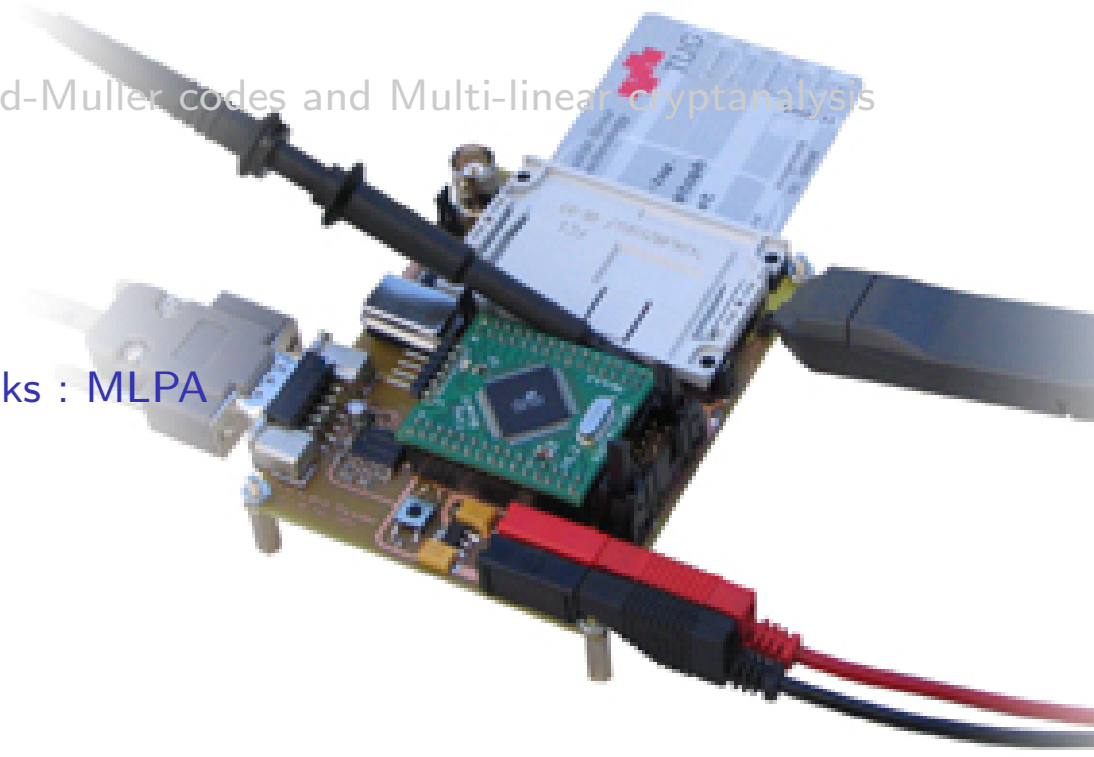
List Decoding in $RM(1, m)$ vs Piling-Up Lemma

Using a list decoding algorithm in $RM(1, m)$,

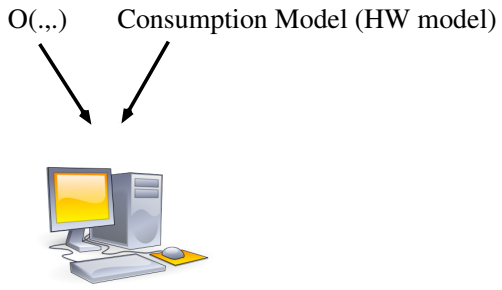
- we do not have to rely on the S-Boxes independence assumption.
- we do not have to know the structure of the target function (seen as a black box).

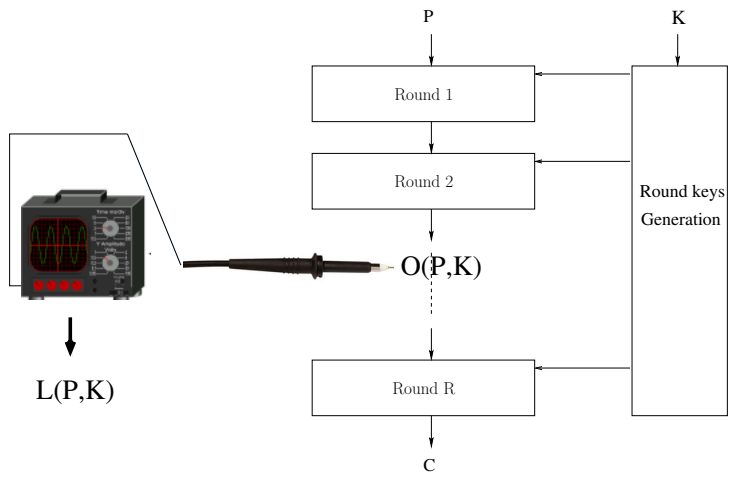
However, the piling-up lemma allows to find approximations with very small biases.

Plan



Side channel measurements





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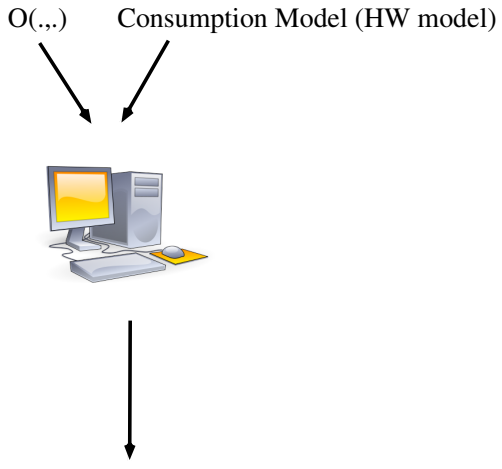

$$
\begin{array}{|l|l|}
\hline
\epsilon_1 & <\gamma_1, K>=<\alpha_1, P>+<\beta_1, HW(O(P,K))> \\
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\epsilon_3 & <\gamma_3, K>=<\alpha_3, P>+<\beta_3, HW(O(P,K))> \\
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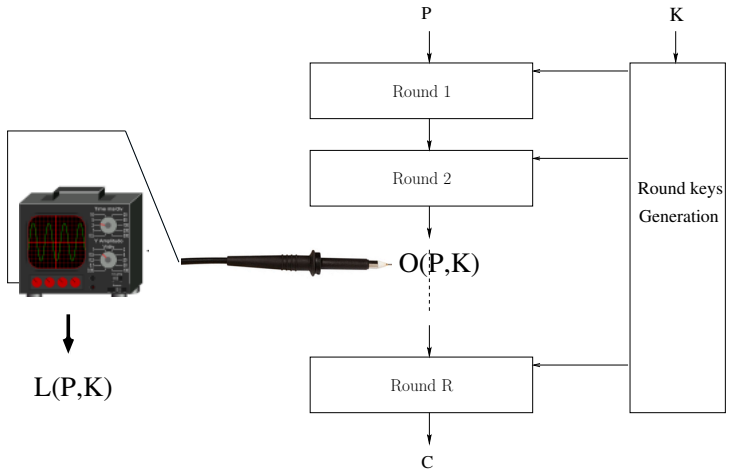
 & \cdots\cdots\cdots \\
\epsilon_n & <\gamma_n, K>=<\alpha_n, P>+<\beta_n, HW(O(P,K))> \\
\hline
\end{array}
$$



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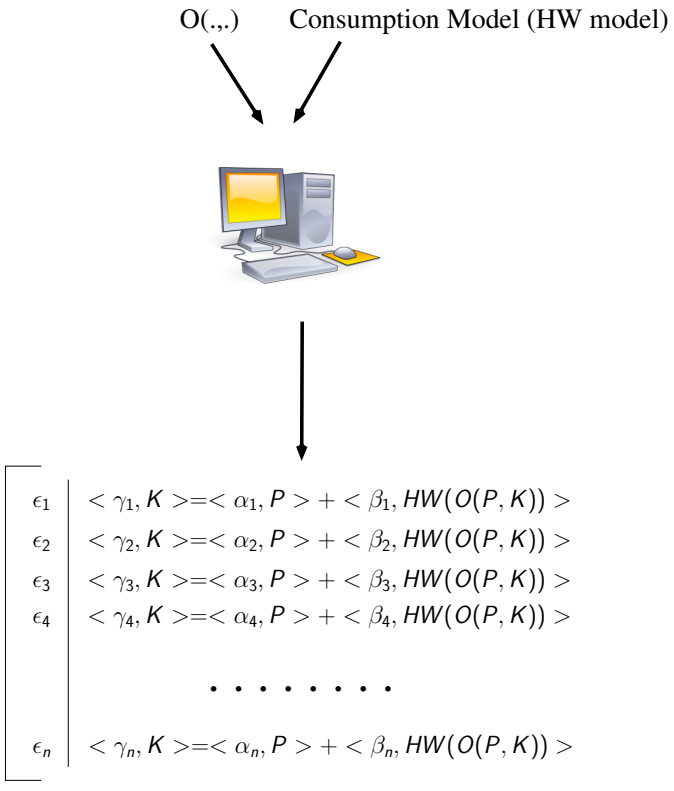

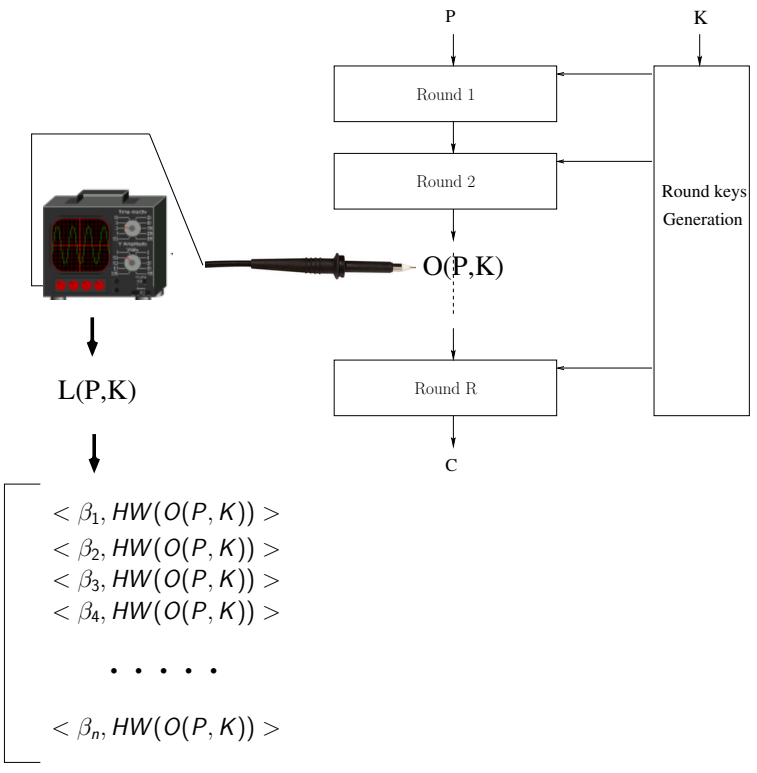

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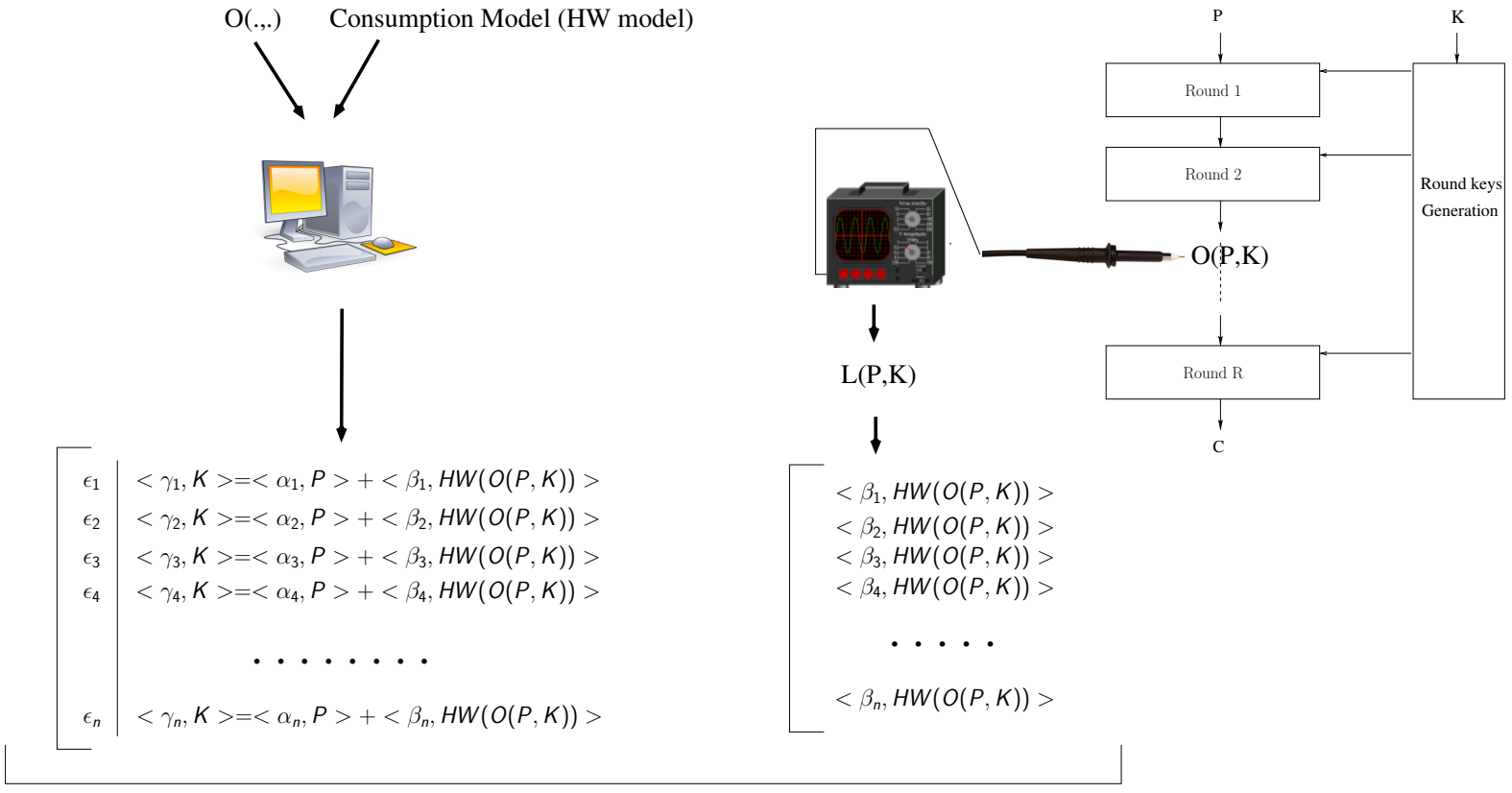

Get a subpart of K by solving the linear system

Hamming weight approximation : DES implementation

From traces "secmatv1_2006_04_0809"
http://www.dpacontest.org/

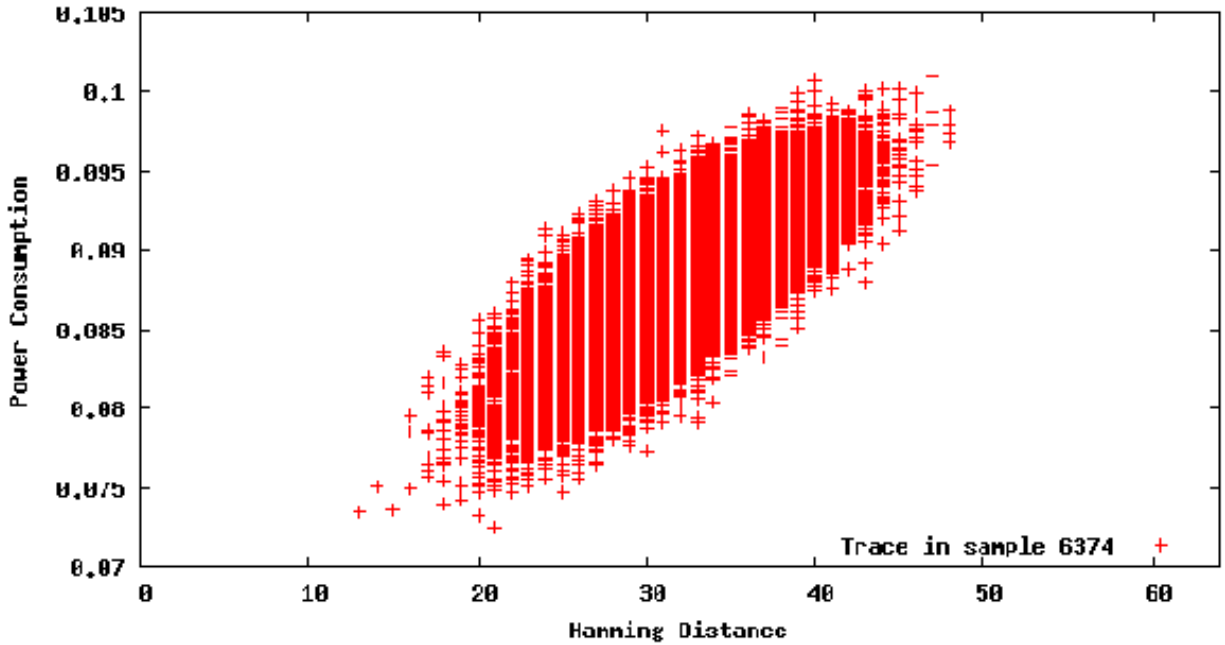

FIG.: Repartition of power consumption with respect to the Hamming Distance model

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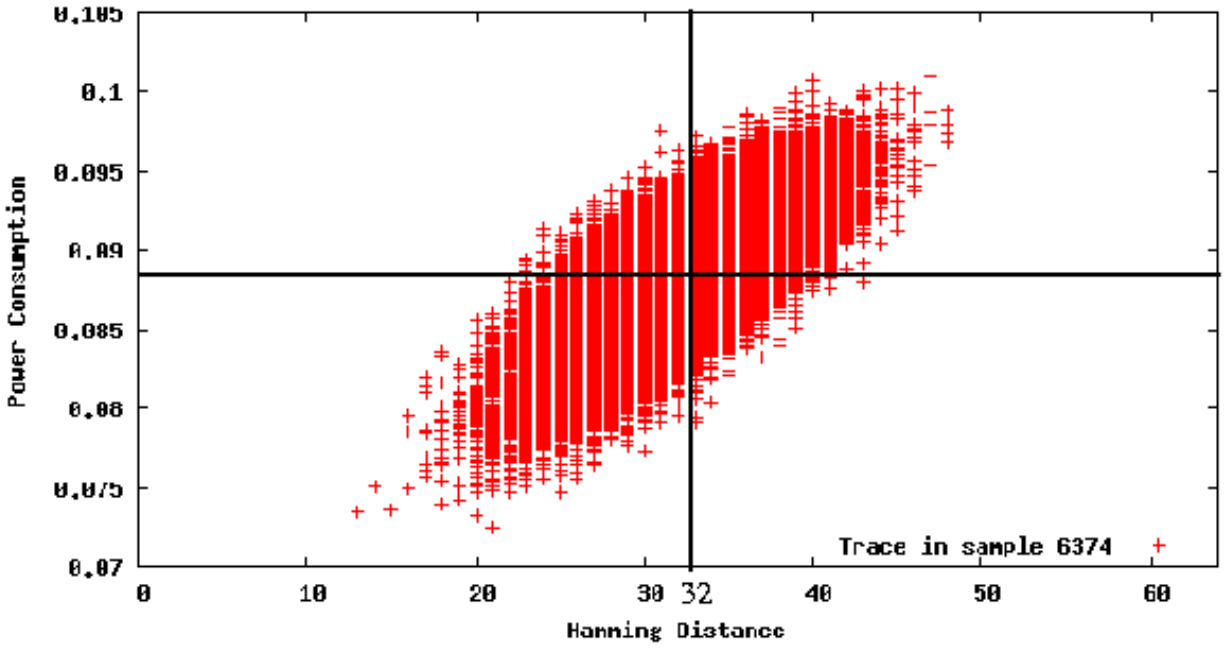


FIG.: Repartition of power consumption with respect to the Hamming Distance model

Attack algorithm and first results on DPA-contest traces

1. Offline static computation : Find many and good approximations of the intermediate data Hamming weight (for every output mask).
2. Online attack : multi-linear cryptanalysis assuming "Leaked information = Hamming distance".

Cipher	rounds	# linear equ.	# key bits	# traces
DES	1	533	40	500
DES	2	1452	27	500
DES	15	488	41	500
DES	16	446	37	500

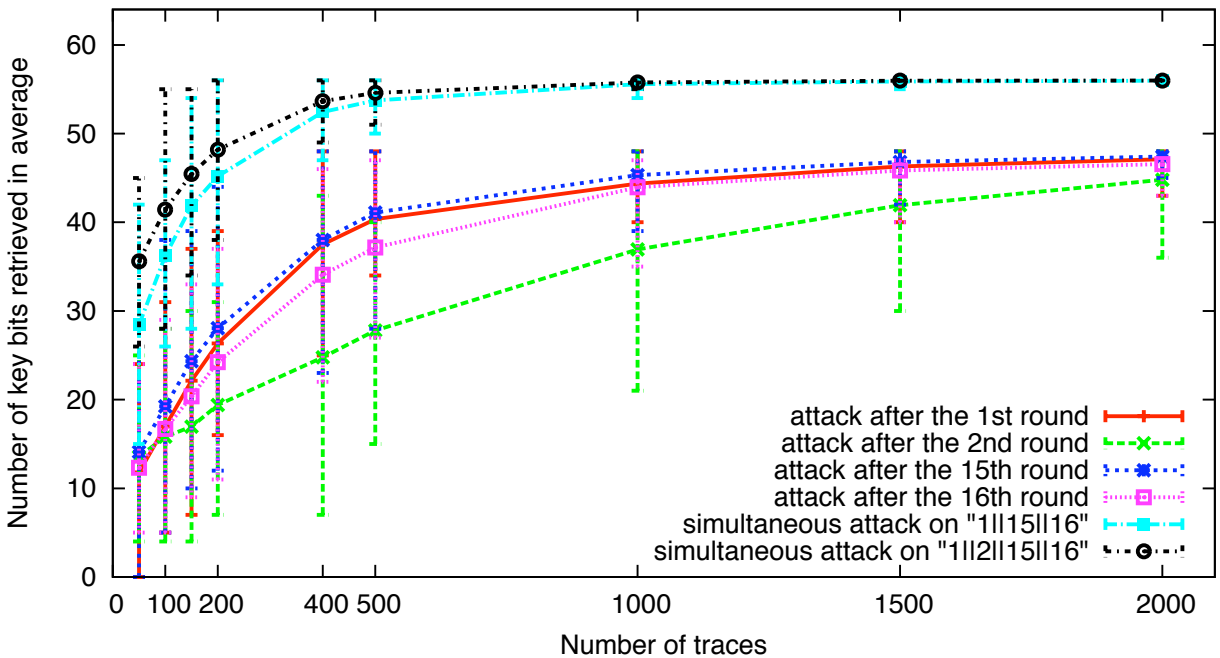
Approximation examples

Output Mask β (in binary) : 100000
Taken on the hamming weight of the bit flips after two rounds.

Bias	Equations (plain part)	Equations (key part)
0.0215	1 + P[5, 26, 27, 31, 45, 53, 61]+	K[6, 7, 29, 38, 52]
0.0134	0 + P[28, 29, 31, 37, 45, 53]+	K[6, 7, 29, 61]
0.0156	1 + P[5, 28, 29, 31, 37, 45]+	K[6, 29, 38, 61]
0.0189	1 + P[5, 28, 29, 31, 37, 53]+	K[7, 29, 38, 61]
0.0163	0 + P[5, 8, 9, 37, 45, 53, 61]+	K[6, 7, 38, 52, 61]
0.0223	0 + P[5, 14, 15, 31, 37, 45, 61]+	K[6, 29, 38, 52, 61]
0.0182	0 + P[5, 28, 29, 31, 37, 53, 61]+	K[7, 29, 38, 52, 61]
0.0157	1 + P[5, 26, 27, 31, 37, 53, 61]+	K[7, 29, 38, 52, 61]
0.0191	0 + P[5, 26, 27, 31, 37, 45, 53, 61]+	K[6, 7, 29, 38, 52, 61]

Simultaneous attack results on DPA-contest traces

Cipher	rounds	# linear equ.	# key bits	# traces
DES	1||15||16	1467	36	100
DES	1||15||16	1467	53	500
DES	1||2||15||16	2919	41	100
DES	1||2||15||16	2919	54	500



Training device attack

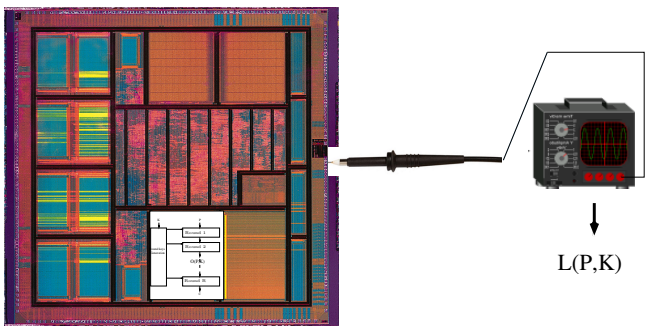
Remark

The List-decoding algorithm operates on the target boolean function as a black-box.

Getting the linear approximations from a twin board
i.e. Chosen plaintexts and keys

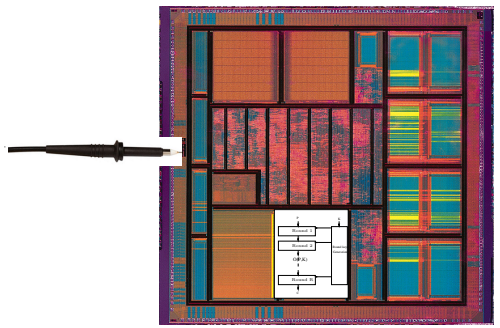
- Approximations directly linked to the leaked information.
  much more accurate.
- No need to choose a power consumption model.
- No need to know the target block cipher.

A template-like attack



Training Board

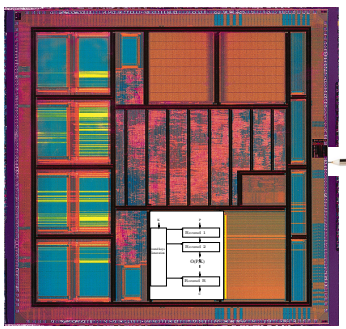
Chosen plaintext
Chosen Key

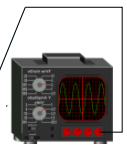


Target Board

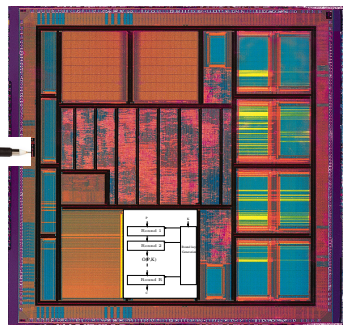
Known plaintext
Secret Key

A template-like attack





L(P,K) Consumption Model (HW model)



Training Board

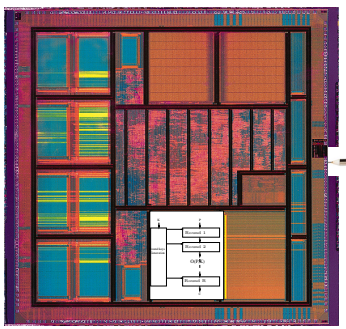
Chosen plaintext
Chosen Key

Target Board

Known plaintext
Secret Key

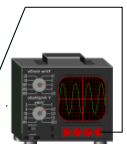
$$
\begin{array}{|c|l|}
\hline
\epsilon_1 & <\gamma_1, K>=<\alpha_1, P>+<\beta_1, L(P,K)> \\
\epsilon_2 & <\gamma_2, K>=<\alpha_2, P>+<\beta_2, L(P,K)> \\
\epsilon_3 & <\gamma_3, K>=<\alpha_3, P>+<\beta_3, L(P,K)> \\
\epsilon_4 & <\gamma_4, K>=<\alpha_4, P>+<\beta_4, L(P,K)> \\
 & \cdots\cdots\cdots \\
\epsilon_n & <\gamma_n, K>=<\alpha_n, P>+<\beta_n, L(P,K)> \\
\hline
\end{array}
$$

A template-like attack



Training Board

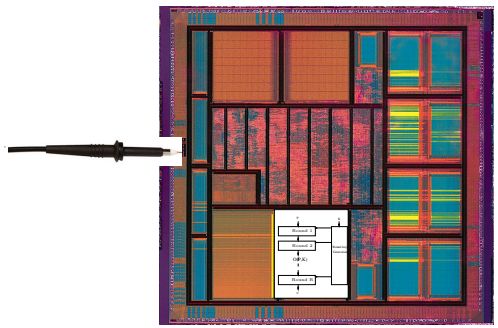
Chosen plaintext
Chosen Key



L(P,K)

Consumption Model (HW model)

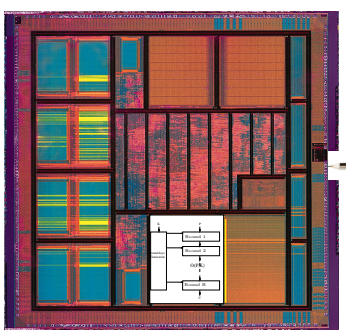
$$\begin{array}{|c|l|}
\epsilon_1 & <\gamma_1, K>=<\alpha_1, P>+<\beta_1, L(P,K)> \\
\epsilon_2 & <\gamma_2, K>=<\alpha_2, P>+<\beta_2, L(P,K)> \\
\epsilon_3 & <\gamma_3, K>=<\alpha_3, P>+<\beta_3, L(P,K)> \\
\epsilon_4 & <\gamma_4, K>=<\alpha_4, P>+<\beta_4, L(P,K)> \\
 & \cdots\cdots\cdots \\
\epsilon_n & <\gamma_n, K>=<\alpha_n, P>+<\beta_n, L(P,K)>
\end{array}$$



Target Board

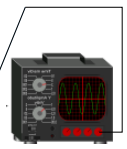
Known plaintext
Secret Key

A template-like attack



Training Board

Chosen plaintext
Chosen Key

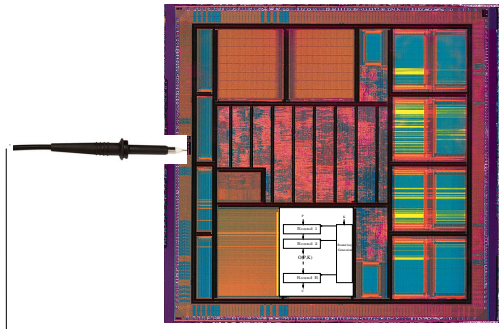


L(P,K)

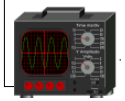
Consumption Model (HW model)

$$\begin{array}{|c|l|}
\epsilon_1 & <\gamma_1, K>=<\alpha_1, P>+<\beta_1, L(P,K)> \\
\epsilon_2 & <\gamma_2, K>=<\alpha_2, P>+<\beta_2, L(P,K)> \\
\epsilon_3 & <\gamma_3, K>=<\alpha_3, P>+<\beta_3, L(P,K)> \\
\epsilon_4 & <\gamma_4, K>=<\alpha_4, P>+<\beta_4, L(P,K)> \\
 & \cdots\cdots\cdots \\
\epsilon_n & <\gamma_n, K>=<\alpha_n, P>+<\beta_n, L(P,K)>
\end{array}$$

Get a subpart of K by solving the linear system



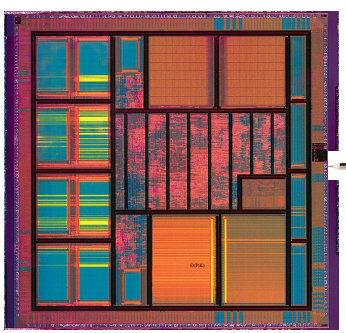
Target Board

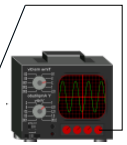


Known plaintext
Secret Key

L(P,K)

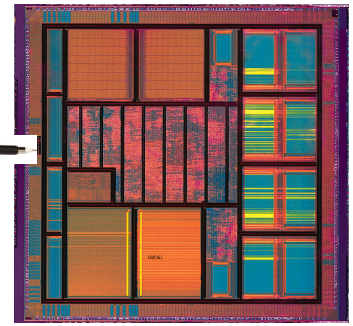
A template-like attack





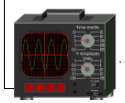
L(P,K)

Consumption Model (HW model)



Target Board

Known plaintext
Secret Key



Training Board

Chosen plaintext
Chosen Key

$$\begin{array}{l|l} \epsilon_1 & <\gamma_1, K>=<\alpha_1, P>+<\beta_1, L(P,K)> \\ \epsilon_2 & <\gamma_2, K>=<\alpha_2, P>+<\beta_2, L(P,K)> \\ \epsilon_3 & <\gamma_3, K>=<\alpha_3, P>+<\beta_3, L(P,K)> \\ \epsilon_4 & <\gamma_4, K>=<\alpha_4, P>+<\beta_4, L(P,K)> \\ & \cdots\cdots\cdots \\ \epsilon_n & <\gamma_n, K>=<\alpha_n, P>+<\beta_n, L(P,K)> \end{array}$$

L(P,K)

Get a subpart of K by solving the linear system

Next Steps on MLPA :

- MLPA on a consumption model refinement.
- HO-MLPA.
- MLPA on masked implementation.
- Other block ciphers.
- Better or More linear approximations.
- MLPA template attack.
- Unknown block cipher attack.
- *Higher degree approximations.*

The end.

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